On the relative numbers of Star Images photographed in different parts of the Plates for the Oxford portion of the Astrographic Catalogue. Second Paper. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. In *Monthly Notices*, lxii. p. 434, a preliminary discussion was given of the counts of star images in different parts of plates taken for the *Astrographic Catalogue*; and it was shown that the star density was greatest at a certain distance (r_0) from the centre of the plate and fell off rapidly outside this distance; the value of r_0 for different object-glasses being different,—*e.g.* for Oxford $r_0 = 33'$, for Paris $r_0 = 58'$, for Algiers $r_0 = 49'$, for Toulouse $r_0 = 30'$.

The natural inference is that the value of r_0 depends on the position of the plate, increasing as the plate is pushed further within the focus.

Similar results were found for the Potsdam telescope, with $r_0 = 28'$ (*Astron. Nachrichten*, No. 3817); and for the Greenwich telescope with $r_0 = 38'$ (*Greenwich Astrographic Catalogue*, vol. i., pp. vi and vii). On the other hand, the late M. Loewy has questioned the reality of the phenomenon in his annual report of the Paris Observatory for 1905, in the following terms:—

"D'autres expériences ont eu pour objet de déterminer jusqu'à quel point une même carte pouvait être considérée comme homogène au point de vue des grandeurs. On sait que M. H. H. Turner a été conduit à penser que cette homogénéité laissait à désirer, en faisant, pour un grand nombre d'épreuves publiées, le relevé des nombres d'étoiles qui tombent dans les divers carrés du réseau. Pour contrôler ce résultat par une autre voie, on a photographié un groupe d'étoiles avec des poses, décroissantes et de légers déplacements systématiques, afin d'obtenir des images faciles à identifier, et en même temps à la limite de visibilité. On a ensuite répété l'opération en imprimant à la lunette d'autres déplacements systématiques de manière à donner le même temps de pose aux mêmes étoiles dans diverses parties de la plaque. On a pu ainsi constater que la visibilité des images relatives aux astres les plus faibles est la même dans toutes les régions du champ. Le defaut signalé par M. Turner, et qui a pu se produire à la suite d'une mise au foyer défectueuse, ne semble donc pas très redoutable dans les circonstances actuelles" (p. 10).

One possible interpretation of the discrepancy between M. Loewy's conclusion and those above quoted is that his test was not so delicate a one as the numerical test elsewhere applied. The results obtained by counting star images are consistent and definite, and clearly afford a satisfactory measure of something. The fact that other methods fail to detect that something may therefore only enhance the value of the method which reveals it: there are almost certain to be uses to which a new test can be applied, even though they may not be foreseen at the moment. The quantity

measured has reference to the focal length of the instrument, and therefore to the scale of the photograph; and it is at any rate of interest to see how far such measures throw light on variations of focal length with time or with temperature; or on the inclination of the plate to the axis of the telescope (which may be regarded as variation in focal length for different parts of the plate).

2. Three volumes of the *Oxford Astrographic Catalogue* have now been published, containing measures of $3 \times 160 = 480$ plates, with centres in zones $+31^\circ$, $+30^\circ$, $+29^\circ$. Generous volunteer assistance

A B

2	10	26	50	82	122	170	226	290	362	442	530	626	8
10	18	34	58	90	130	178	234	298	370	450	538	634	16
26	34	50	74	106	146	194	250	314	386	466	554	650	24
50	58	74	98	130	170	218	274	338	410	490	578	674	32
82	90	106	130	162	202	250	306	370	442	522	610	706	40
122	130	146	170	202	242	290	346	410	482	562	650	746	48
170	178	194	218	250	290	338	394	458	530	610	698	794	56
226	234	250	274	306	346	394	450	514	586	666	754	850	64
290	298	314	338	370	410	458	514	578	650	730	818	914	72
362	370	386	410	442	482	530	586	650	722	802	890	986	80
442	450	466	490	522	562	610	666	730	802	882	970	1066	88
530	538	554	578	610	650	698	754	818	890	970	1058	1154	96
626	634	650	674	706	746	794	850	914	986	1066	1154	1250	

C D

FIG. 1.

from Miss Riddle has enabled us to count the measures made on these plates on a systematic plan arranged so as to exhibit such variations in focal length. It seemed desirable to discuss these measures in order to decide whether to extend the work to the other four volumes as they appear; and to give a brief summary of the discussion, in order to enable our colleagues to judge of the expediency of undertaking similar work elsewhere.

3. Each plate was divided into four quarters, which will be denoted by XY, xY, Xy, xy. The capital letters refer to large values of x or y,—*i.e.* values exceeding 13·000; the small letters

to values less than 13·000. An alternative designation would thus be

$$XY = NE \quad xY = NW \quad Xy = SE \quad xy = SW$$

Each quarter was then divided up into ten regions, the average distances of which from the plate centre were in the ratios 1, 2, 3 .... 10, approximately, the boundaries of the regions following the reseau lines. The dividing lines for one of the quarters are shown in fig. 1. A represents the plate centre and D the corner of the reseau. Representing a side of the reseau by 2, the middle points of the reseau squares along A B will be at distances 1, 3, 5 .... 25. It is readily seen that on this scale the numbers written in the squares represent $r^2 = x^2 + y^2$, where x and y are the co-ordinates of the centre of the square referred to A as origin. Now if we take ten regions at distances t, $2t$, $3t$ .... $10t$ from A as centre, the value for the outermost will be $100t^2$; and a glance at fig. 1 shows that we must put approximately $100t^2 = 1100$, or t^2 = about 11.

Table I.

Letter for Region.	No. of Squares.	Average Value of r^2.	t^2.	r in Minutes of Arc.
a	3	7·3	7·3	7
b	5	28	7·0	13
c	14	76	8·4	22
d	15	150	9·4	31
e	23	248	9·9	39
f	23	363	10·1	48
g	34	508	10·4	56
h	29	673	10·5	65
i	15	864	10·7	73
k	8	1090	10·9	82

The thick lines were drawn with this value of t^2 in mind. The 4th and 5th columns of Table I. show that the grouping might have been better, but this was not noticed until so much work had been done that it seemed better to keep the system unaltered. It is not essential to have the distances in exact arithmetical progression.

4. The counts were then treated as shown in the following example:—

Table II.

Total counts in the regions for Plates 2034–2057, *Zone* +31°.

No. squares =	3	5	14	15	23	23	34	29	15	8	169
Region =	*a.*	*b.*	*c.*	*d.*	*e.*	*f.*	*g.*	*h.*	*i.*	*k.*	Total.
XY	25	30	101	137	222	276	372	297	132	64	1656
xY	22	42	124	136	266	228	332	247	97	36	1530
Xy	21	38	106	113	226	205	340	312	112	62	1535
xy	22	38	89	120	193	208	315	236	89	35	1345
	90	148	420	506	907	917	1359	1092	430	197	6066

The number of squares in a region varies considerably, as shown in the top line. The next procedure is to divide by this number, so as to get the average star density in each region. Then to reduce this density to a uniform scale for different batches of plates, the quotients are again divided by 6066/4, the average total number of stars on a single quarter-plate. The intermediate results need not be given here; in Table III. are shown the final values obtained for star density in the different regions.

TABLE III.

Star Density on a uniform scale deduced from Table II.

	a.	*b.*	*c.*	*d.*	*e.*	*f.*	*g.*	*h.*	*i.*	*k.*	Total.
XY	55	40	48	61	64	79	72	68	58	53	598
*x*Y	48	56	59	60	77	65	65	56	43	30	559
X*y*	46	50	50	50	65	59	66	72	50	52	560
xy	48	50	42	53	56	60	61	54	39	29	492
Mean	49	49	50	56	65	66	66	63	47	41	552

5. Now the characteristic feature of the variation in density is the gradual rise up to about f and g and the fall afterwards. The maximum represents the position of best focus, and it is important to note any variations in this position. For this purpose it is convenient to have a single number of some kind rather than a system of 10, and the indicator adopted for a preliminary investigation is

$$\Delta = (c + d + e) - (g + h + i).$$

It will be seen that an increase in Δ means that the maximum comes nearer a,—*i.e.* nearer the plate centre,—and presumably that the plate is further from the object-glass: a decrease in Δ corresponds to the plate being pushed in nearer the O.G.

To get an idea of the physical meaning of Δ, let us take the theoretical formula for star density on a plate given in *Mon. Not.*, lxii. p. 441, viz.—

$$\text{Density} = a\{1 \backsim c(r^2 - r_0^2)\}$$

where r is the distance of the point on the plate from the plate centre; r_0 is the distance of the maximum from the centre; and $c = 10^{-4}$ if r and r_0 are expressed in minutes of arc. We may put r_0 successively equal to the selected distances in Table I.; and take $a = 66$, which is the mean maximum, on the system adopted in Table III.; and we then get the densities as in Table IV.

TABLE IV.

Theoretical Star Densities, with different positions of the maximum, and corresponding values of Δ and n.

r_0.	a.	b.	c.	d.	e.	f.	g.	h.	i.	k.	Δ.	n.
7′	66	65	63	60	56	51	45	38	31	22	+65	8039
13	65	66	64	61	57	52	46	39	32	23	+65	8202
22	63	64	66	63	59	54	48	41	34	25	+65	8508
31	60	61	63	66	62	57	51	44	37	28	+59	8883
39	56	57	59	62	66	61	55	48	41	32	+43	9263
48	51	52	54	57	61	66	60	53	46	37	+13	9508
56	45	46	48	51	55	60	66	59	52	43	−23	9526
65	38	39	41	44	48	53	59	66	59	50	−51	9071
73	31	32	34	37	41	46	52	59	66	57	−65	8210
82	22	23	25	28	32	37	43	50	57	66	−65	6833
No. of squares	3	5	14	15	23	23	34	29	15	8		

6. The values of Δ are shown in the last column but one of Table IV. When r_0 is small or large,—*i.e.* when the best focus is very near the centre or the corner of the plate,—Δ is constant, because the distances represented by $c\ d\ .\ .\ .\ .\ i$ are all on one side of the maximum, and hence $\Delta \equiv (c+d+e)-(g+h+i)$ does not change. But such cases do not occur in practice, the plate being always focussed for some distance from the centre, and then Δ is a convenient index to the position of best focus. In the last column is given, n, the total number of stars on the quarter-plate, obtained by assuming the density constant over each of the regions $a, b\ .\ .\ .\ .\ k$; so that the number of stars in each region is the product of the density by the number of squares in the region as shown in the last line of the table. The values of n so found imply the further assumption that the maximum density is the same (viz. 66 per square) whatever be the position of the plate. This assumption is no doubt incorrect, but it may not be far from the truth. The optical image of a star in focus near the axis of the lens is almost certainly smaller than the best focussed image of a star near the edge of the field. Hence, if the number of stars photographed depended on the size of best image alone, the maximum density would fall off as we left the centre of the field, and might fall off even rapidly. But other factors influence the photographic star image—atmospheric tremor, errors of driving and guiding, etc.—which are independent of the distance from the centre of the field, and the maximum density may not diminish so rapidly as might at first appear. It seems worth while to examine the consequences of assuming it to be constant, at any rate in the first instance. But we shall return to this point later. Our first business is with the *position* of the maximum, as indicated by the quantity Δ.

7. We proceed to tabulate the values of Δ for the four quarters,

arranging all the plates of vols. i., ii., iii. of the Oxford Catalogue in groups, as sufficiently indicated by columns 1, 2, 3, 4 in Table V. The detailed grouping was determined by considerations which need not concern us here; nor need we, in a preliminary survey, pay much attention to the fact that one or two of the individual groups are small either in number of plates or number of stars.

TABLE V.

Mean Values of Δ for Plates in vols. i., ii., iii. *of the Oxford Astrographic Catalogue.*

Zone.	Limiting Nos.	No. of Plates.	No. of Stars.	Values of Δ. XY.	*x*Y.	X*y*.	*xy*.	Whole.
29°	525– 542	16	8099	+48	+37	+42	+59	+46
29	700– 855	14	8589	+27	+34	+10	+28	+25
30	896– 921	15	4293	+25	+42	+51	+17	+34
29	937–1023	32	7300	+51	+49	+52	+15	+42
29	1236–1486	18	7913	+52	+12	+23	+37	+31
Totals and Means		95	36194	+41	+35	+36	+31	+36
30	1564–1585	11	12593	−28	− 5	−44	+15	−16
30	1586–1597	12	5933	−49	− 4	−34	+30	−14
31	1598–1606	8	4877	−76	+ 7	−81	+ 3	−37
Totals and Means		31	23403	−51	− 1	−53	+16	−22
29	1747–1843	20	6706	− 1	+51	−20	+34	+16
31	1834–1849	7	6245	−37	+ 3	−38	−21	−23
29	1850–1900	26	8860	+ 2	+37	−30	+34	+11
30	1865–1915	14	8178	− 6	+38	−30	+27	+ 7
30	1916–1943	26	5989	− 1	+48	−22	+38	+16
Totals and Means		93	35978	− 9	+35	−28	+22	+ 5
30	1946–2060	26	11217	−30	+32	−56	+ 8	−11
31	1973–1996	11	4006	−22	+30	−59	−11	−16
31	1997–2032	26	19984	−29	+37	−64	+ 1	−14
31	2034–2057	24	6066	−25	+32	−23	− 3	− 5
Totals and Means		87	41273	−27	+33	−50	− 1	−11
30	2061–2101	26	7797	+ 1	+12	−30	+42	+ 6
31	2087–2155	15	4513	−39	+46	− 9	+34	+ 8
30	2102–2111	10	2693	0	+29	−27	+17	+ 5
30	2136–2196	16	6726	−13	+25	−47	+26	− 2
31	2202–2234	15	4445	−14	+24	− 1	+40	+12
30	2217–2482	4	1163	+12	−16	−28	+56	+ 6
31	2235–2271	26	6095	− 5	+47	−22	+29	+12
31	2274–2299	12	4871	−33	− 6	−41	+25	−14
Totals and Means		124	38303	−11	+20	−26	+34	+ 4
31	2311–2349	18	4596	+ 2	+21	− 5	+43	+15
29	2450–2500	30	11874	−23	+25	−22	+23	+ 1

8. The reasons for separation at the places indicated by the horizontal lines are as follows:—

(*a*) Between plate 1557 and 1558 (summer of 1900) the new dome was erected. The object-glass was dismounted and the eye end also.
(*b*) After plate 1606, the instrument was devoted to the Eros work. The eye end was taken off to study possible instrumental adaptations.
(*c*) After plate 1943 the O.G. was dismounted for cleaning.
(*d*) Some change seems to have taken place between 2057–2061, or near this date, though nothing is recorded in explanation.
(*e*) At plate 2300 the eye end was dismounted to study adaptation to photographs of the Moon among the stars.
(*f*) After plate 2349 the O.G. was dismounted and taken to Egypt for the eclipse of 1905.

There was apparently a slight change of focal length at each of these epochs, due to accidental causes. It is curious that, on the whole, the value of Δ should show an *increase* from the second group onwards, for this corresponds to the plate being further from the O.G., and not nearer to it, as might have been explained by the gradual wearing of the studs against which it is pressed. No change was made deliberately in the adjustment of these until after all the plates had been taken.

9. It is a reasonable assumption that on each occasion the focal length was changed without disturbing the relationship of the quarters to each other. Hence we may subtract the value of Δ for the whole plate (given in the last column) from each quarter. It will be sufficient here to give the means of groups.

Table VI.

Values of Δ for each quarter relative to mean Δ for Plate.

	Limiting Nos.	Weight.	XY.	xY.	Xy.	xy.
I.	525–1486	5	+ 5	− 1	0	− 5
II.	1564–1606	3	−29	+21	−31	+38
III.	1747–1943	5	−14	+30	−33	+17
IV.	1946–2057	5	−16	+44	−39	+10
V.	2061–2299	5	−15	+16	−30	+30
VI.	2311–2349	1	−13	+ 6	−20	+28
VII.	2450–2500	2	−24	+24	−23	+22

There is a decided change after the first group. In the first group the quarters XY and Xy are slightly positive, corresponding to the X side of the plate (large R.A.*s*) being a little further from the O.G. than the x side. In all the later groups the reverse is the

case, and the relationship remains fairly constant. It is true there seems to be a slight diminution in xY and an increase in Xy, but the appearance is chiefly due to the last two groups, which have small weight.

10. The changes in general focal length seem more striking than any relative change. But it is possible to take another view of these changes. Are they possibly seasonal? No deliberate change in focus was made at any of the epochs above mentioned, and at one of them no interference with the instrument is recorded.

11. Let us try the extreme assumption that they are all due to temperature or other seasonal change. In the third column of Table VII. is given the month in which the majority of the plates in the group were taken. Arranging the groups according to these months we have—

Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
+6	+7	−2	+16	+12	−11	+1	−16	−14		−5	
	+8		+16		−16		−14	−37		+11	+15
	+5		+12		−14			−23			
			+6								

There certainly seems to be something to be said for a seasonal effect. The values of Δ from December to May are all positive with one slight exception, and from June to November all negative. If we express the effect as a simple harmonic term, with a constant added, we should have figures something like those given below:—

Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
+4	+10	+12	+10	+4	−4	−12	−18	−20	−18	−12	−4

Applying these as corrections with reversed sign to the individual groups, we get the results of Table VII.

TABLE VII.

Possible seasonal change in Δ.

	Uncorrected Δ.	Month.	Correction.	Corrected Δ.
1564–1585	−16	Aug.	+18	+2
1586–1597	−14	Sept.	+20	+6
1598–1606	−37	Sept.	+20	−17
Mean	−22	...	+19	−3
1747–1843	+16	Apr.	−10	+6
1834–1849	−23	Sept.	+20	−3
1850–1900	+11	Nov.	+12	+23
1865–1915	+7	Feb.	−10	−3
1916–1943	+16	Apr.	−10	+6
	+5	...	+1	+6

TABLE VII.—*continued.*

Possible seasonal change in Δ.

	Uncorrected Δ.	Month.	Correction.	Corrected Δ.
1946–2060	− 11	June	+ 4	− 7
1973–1996	− 16	June	+ 4	− 12
1997–2032	− 14	Aug.	+ 18	+ 4
2034–2057	− 5	Nov.	+ 12	+ 7
	− 11	...	+ 10	− 2
2061–2101	+ 6	Jan.	− 4	+ 2
2087–2155	+ 8	Feb.	− 10	− 2
2102–2111	+ 5	Feb.	− 10	− 5
2136–2196	− 2	Mar.	− 12	− 14
2202–2234	+ 12	Apr.	− 10	+ 2
2217–2482	+ 6	Apr.	− 10	− 4
2235–2271	+ 12	May	− 4	+ 8
2274–2299	− 14	June	+ 4	− 10
	+ 4	...	− 7	− 3
2311–2349	+ 15	Dec.	+ 4	+ 19
2450–2500	+ 1	July	+ 12	+ 13

The figures can thus be explained either by a seasonal change in focal length, or by a series of discontinuities at the times when either eye end or O.G. were dismounted.

Against the latter explanation there is the fact that no discontinuity was recorded about plate No. 2057, so that the seasonal effect is somewhat more probable.

The first set of plates (Nos. 525–1486) throws no light on the matter, for the most divergent groups in the last column of Table IV. are the first two, which were both taken chiefly in September, so that we must set down their difference to accident.

Summarising the results so far, we find—

(α) That the focal position of the plate was essentially different in the two periods before and after the erection of the new dome in 1900.

(β) In the first period the plate was further from the object-glass ($\Delta = +36$ for the mean of the four quarters), and satisfactorily normal to the line of collimation (values of Δ nearly same in all quarters).

(γ) In the second period the plate was nearer the O.G. ($\Delta = 0$ or less), and the X side (large R.A.'s) much nearer than the x side (small R.A.'s). The values of Δ for the two sides are about $\Delta = -25$ and $\Delta = +25$ respectively.

Moreover, there are variations in the focal length which may be seasonal, ranging from $\Delta = -20$ in June–September (plate

nearer O.G.) to $\Delta = +12$ in December–April (plate further from O.G.), though these variations may be due to a series of discontinuities.

The result that the plate is further from the O.G. in cold weather and nearer in warm seems a little strange. The telescope tube is no doubt longer in the warm weather; but we are concerned with a differential effect, depending partly on the expansion of the tube and partly on that of the lenses, which may alter their focal length more than the tube alters, and so give an apparently reversed effect. For this reason we cannot look for confirmation (or otherwise) to the observed scale-value of the plate deduced from measures of star images; for this scale value depends on the expansion of tube and plate, perhaps also on that of the reseau.

12. Let us now examine the effect of these differences on the relative number of stars photographed in the different quarter-plates. We find that we have in plates 1564–2500 a mass of tolerably homogeneous material with one chief variable, the mean value of Δ for the whole plate, which probably corresponds to the mean distance of the plate from the O.G. Whether the variations in Δ are due to a seasonal effect or to accidental discontinuities need not concern us if we take Δ itself as the independent variable.

Table VIII.

Total number of Stars counted on each Quarter-Plate.

Mean Δ.	Plate Nos.	Zone.	XY.	xY.	Xy.	xy.
−37	1598–1606	31°	1320	1208	1269	1080
−23	1834–1849	31	1528	1571	1582	1564
−16	1564–1585	30	3175	3332	3161	2925
−16	1973–1996	31	1022	965	1070	949
			7045	7076	7082	6518
−14	1586–1597	30	1582	1540	1462	1349
−14	1997–2032	31	5201	5075	5000	4708
−14	2274–2299	31	1289	1187	1266	1129
−11	1946–2060	30	2951	2743	2901	2622
− 5	2034–2057	31	1656	1530	1535	1345
− 2	2136–2196	30	1820	1525	1813	1568
+ 1	2450–2500	29	3247	3011	2874	2742
			17746	16611	16851	15463
+ 6	2061–2101	30	1972	1891	2019	1915
+ 5	2102–2111	30	752	653	675	613
+ 6	2217–2482	30	314	293	298	258
+11	1850–1900	29	2308	2190	2250	2108
+ 7	1865–1915	30	2290	2023	2101	1764
+ 8	2087–2155	31	1193	1174	1130	1006
			8829	8224	8473	7664

TABLE VIII.—*continued.*

Total number of Stars counted on each Quarter-Plate.

Mean Δ.	Plate Nos.	Zone.	XY.	xY.	Xy.	xy.
+12	2235–2271	31°	1560	1494	1598	1443
+12	2202–2234	31	1185	1103	1147	1010
+16	1747–1843	29	1679	1630	1755	1642
+15	2311–2349	31	1188	1194	1099	1115
+16	1916–1943	30	1647	1449	1527	1366
			7259	6870	7126	6576

Reducing the numbers to percentages we find for the means of groups—

TABLE IX.

Mean Δ.	XY.	xY.	Xy.	xy.
−23	101·6	102·1	102·2	94·1
− 8	106·4	99·7	101·1	92·8
+ 7	106·4	99·1	102·0	92·3
+14	104·3	98·7	102·4	94·5
Mean	104·7	99·9	101·9	93·4

13. We have now to correct these numbers for the changes in Δ. Table IV. gives in the last two columns the values of n (the whole number of stars on a quarter-plate) for different values of Δ on certain assumptions specified in § 6. We may make a small table of the *percentage* correction to n as follows :—

Δ =	+60	+50	+40	+30	+20	+10	0	−10	−20	−30	−40	−50	−60
Corrn. to n p. c. =	+7·5	+4·5	+2·8	+1·7	+0·9	+0·3	+0·1	0·0	+0·1	+1·0	+2·5	+4·5	+9·7

Now the mean values of Δ for the groups of Table IX. and for each quarter-plate, and the consequent corrections to percentage, would be as follows :—

TABLE X.

Group (Table IX.)	Mean Value of Δ.				Correction to Percentage.				Mean.
	XY.	xY.	Xy.	xy.	XY.	xY.	Xy.	xy.	
I.	−41	+ 9	−55	− 4	+2·7	+0·2	+7·1	0·0	+2·5
II.	−27	+20	−41	+13	+0·7	+0·9	+2·7	+0·4	+1·2
III.	− 5	+24	−26	+34	0·0	+1·2	+0·5	+2·1	+0·9
IV.	− 4	+38	−14	+36	0·0	+2·7	0·0	+2·4	+1·3

Subtracting the mean shown in the last column from the separate corrections for each quarter-plate (so as to keep the mean for the whole-plate zero), the numbers of Table IX. corrected would then be as in Table XI.

TABLE XI.

Table IX. *corrected to uniform* $\Delta = -10$.

	XY.	xY.	Xy.	xy.
	101·8	99·8	106·8	91·6
	105·9	99·4	102·6	92·0
	105·5	99·4	101·6	93·5
	102·9	100·1	101·2	95·5
Mean	104·0	99·7	103·1	93·2

14. It is clear that we have not removed the chief part of the differences shown in Table IX. by this process, and we must look to other causes for them. One such cause is undoubtedly *tilt* of

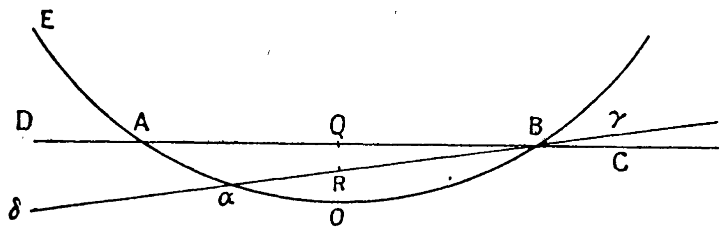

Fig. 2.

the plate, of which no account has yet been taken in dealing with the number of stars as distinct from the position of best focus. That the tilt seriously affects the number of stars can be seen from elementary geometrical considerations. Let E A O B (fig 2) represent a section of the curved field and D A B C a position of the plate normal to the axis.

Now if the plate be tilted to the position $\delta \alpha B \gamma$, it is clear that every portion of the half Q C or $R\gamma$ is brought *closer* to the curve of good focus, and this side of the plate will thus contain more stars all over. On the other side, regarding the plate as first moved parallel to itself to cut the curve at α, and then tilted, we have in what precedes taken account of the movement of translation, but not of the tilt; and the tilt clearly moves every part of this other half-plate *further* from the curve of good focus, so that we lose stars all over it. The effect of tilt is thus greater than that of translation, because by moving the plate parallel to itself we gain in one part and lose in another, whereas by tilt we either gain or lose all over the half-plate. We must qualify this statement a little when we extend the argument to two dimensions, so as to deal with a curved surface instead of the curved arc shown in the figure. But the general nature of the phenomenon remains; and we see that *when a plate is tilted, the side nearer the O.G. gains at the expense of the side more remote.*

15. We thus see how parts of the large differences of Tables IX. and XI. probably arise. The quarter-plates XY and Xy, which have Δ negative, are nearer the O.G. and gain stars

from their opposites xy and xY. But it is not necessary to proceed to a quantitative estimation to see that we cannot in this way explain the whole of the differences. There must be some other contributing cause which makes the Y quadrants 4 per cent. richer in stars than the y quadrants. The difference between the mean Δ for the Y quadrants and the y quadrants is so slight that tilt cannot explain this considerable excess; and, while reserving the study of tilt for a future paper, we may proceed here briefly to consider the possible causes of an excess of stars in the north half of a plate which cannot be explained (so far as can be seen at present) by the position of the plate.

16. The excess of stars in the N half of the plate may be due to any combination of the following causes:—

(α) Optical performance of the O.G., including possible inclination to the line of collimation.

(β) Increased atmospheric absorption for S stars.

(γ) Actual increase in number of stars as we go northwards. When we combine all the plates in a zone, we eliminate (to a large extent) variations of uniformity in R.A., but a change with declination may be persistent.

17. It is possible to estimate the approximate magnitude of cause (β). The formula for atmospheric absorption at Oxford of visual rays is given in *Mem. R.A.S.*, vol. xlvii., as 0·25 sec. Z.D. We may put Z.D. = 25°; and the variation of sec. Z.D. for 1°, which is the distance between N and S halves of the plate, is about 0·01. The correction is thus about ·0025 magnitude. Now Newcomb gives (in his book, *The Stars: a Study of the Universe*, p. 283) the number of stars of different magnitudes as in the second column of Table XII., whence we get the totals of that magnitude and brighter as in the third column, from which we

Table XII.

Mag.	No. of Stars.	Total = N.	log N.	Diff.
6·5	2	2	0·30	
				·30
7·0	2	4	0·60	
				·30
7·5	4	8	0·90	
				·38
8·0	11	19	1·28	
				·25
8·5	15	34	1·53	
				·27
9·0	29	63	1·80	
				·18
9·5	33	96	1·98	
				·15
10·0	39	135	2·13	
				·17
10·5	64	199	2·30	
				·20
11·0	115	314	2·50	

form log N and its differences as in the 4th and 5th columns. It appears that at magnitude 11, log N is increasing at the rate

of about ·4 per magnitude, so that the increase in log N for ·0025 mag. would be about ·001. This would increase 100 stars to 100·23 (since log 100·23 = 1·001), so that a difference of atmospheric absorption indicated by visual observations would only make a difference of 0·23 per cent. between the upper and lower halves of a plate. Photographic absorption would need to be twenty times as large to explain the observed difference, which is not likely. Sir W. Abney emphasises (*Mon. Not.*, xlvii. p. 265) the necessity for knowing the "spectrum value" of the plates employed if we wish to determine the photographed absorption, but the limits between which it varies are not likely to be wider than from one to four times the visual absorption.

18. It occurred to me that we might get some indication of the photographic absorption by comparing in some way the plates taken at different hour-angles; for instance, in the formula given at the heading of each of our plates for determining magnitude from measures of diameters,

$$\text{mag.} = a - b\sqrt{d},$$

the constant a would vary with the atmospheric absorption. But a little examination showed that the differences of Z.D. between plates taken otherwise under the same conditions were too small to afford a trustworthy indication of the phenomenon, which is swamped by a number of other larger variations. A single example will serve to show this. In Table XIII. are shown in the first three columns the Decl. and R.A. of the plate centre and the hour-angle at the middle of the exposure; in the fourth column is given the Z.D. of the centre; and in the fifth the value of the constant a of the above formula. If these values of a are arranged according to Z.D. we find—

No. Plates.	Mean Z.D.	Mean a.
7	35°·4	16·43
6	41·0	16·78

i.e. the paradoxical result that fainter magnitudes (by 0·38 magnitude) are shown on the plates of 6° greater Z.D. Suspicion arose that the person who measured the plate might influence the result; and on collecting the results for the three measurers shown in the sixth column, it was found that their mean values of a were

BG.	EG.	S.
$16{\cdot}84_5$	$16{\cdot}40_5$	$16{\cdot}50_3$

TABLE XIII.

Plates taken on the night of 1903 *October* 30.

Decl.	R.A.	H.A.	Z.D.	*a*.	Measurer.	Corrn.
+31°	3·31	1·39 E	28°	16·2	EG	+ ·2
,,	5·55	3·41 ,,	45	16·5	EG	+ ·2
,,	6· 4	3·30 ,,	43	17·7	BG	− ·2
,,	6·13	3·13 ,,	41	17·0	BG	− ·2
,,	6·22	3· 3 ,,	39	16·3	BG	− ·2
,,	6·58	3·10 ,,	40	16·3	S	+ ·1
,,	7· 7	3· 0 ,,	39	16·9	S	+ ·1
,,	7·25	2·57 ,,	38	16·6	EG	+ ·2
,,	7·34	2·41 ,,	36	16·8	BG	− ·2
,,	8· 1	2·49 ,,	37	16·4	BG	− ·2
,,	8·28	2·55 ,,	38	16·2	EG	+ ·2
,,	8·46	2·39 ,,	36	16·3	S	+ ·1
,,	8·55	2·31 ,,	35	16·5	EG	+ ·2

but on applying corrections as in the last column the 16·43 only became 16·50 and the 16·78 became 16·75, so that the anomaly was not removed. It seems better to determine the photographic absorption by special observations rather than to seek it in the plates already taken, in which its effect was consciously or unconsciously minimised.

19. We may now turn to cause (γ), the distribution of stars in the sky. That the density increases as we go northwards in the Oxford zones is shown, for instance, in Argelander's *Durchmusterung*. His numbers per square degree for zones +28°, +29°, +30°, +31°, are 14·9, 16·2, 16·4, 16·4; so that the differences per cent. between the north half and the south half of plates in zones with centres +29°, +30°, +31°, here discussed, would be 8·1, 1·3, 0·0; mean, 3·1. This is comparable with our 4·0 per cent., and the difference is what we might expect, considering that Argelander only goes to magnitude 10. But it is unsatisfactory that the difference is chiefly in one zone. We may put this down to accidental error, but the evidence is only sufficient to state a case for inquiry. In *Monthly Notices*, vol. lx., plate 2 (opposite p. 16), Mr. Bellamy has given a curve showing the approximate variation in stellar density (for the Oxford zones) with Galactic latitude somewhat as follows:—

TABLE XIV.

Gal. Lat.	0°.		10°.		20°.		30°.		40°.		50°.		60°.		70°.		80°.		90°.
N = No. of stars	800		460		310		230		180		155		135		120		110		105
Differences		340		150		80		50		25		20		15		10		5	
Log N	2·90		2·66		2·49		2·36		2·26		2·19		2·13		2·08		2·04		2·02
Differences		0·36		0·17		0·13		0·10		0·07		0·06		0·05		0·04		0·02	

20. There are two important points lying away from the curve, but we will neglect these in the following brief and general remarks. The Galaxy cuts the parallel of 30° Dec. about $5\frac{1}{2}^h$ and $19\frac{1}{2}^h$.

(A) At these crossing points the changes of stellar density are very large, but *if they are symmetrical* on opposite sides of the Galaxy there should be a rough conpensation. On one side the N half of a plate will be richer, on the other the S half. We must be prepared, however, to find this compensation break down if the Milky Way is unsymmetrical.

(B) In the neighbourhood of 0^h the Oxford zones are near the Milky Way, south of it, and nearly parallel to it. Thus to pass from the southern half of a plate to the north is to approach the Milky Way, and consequently to find more stars. In these R.A.'s, therefore, we may expect to find the main contributions to the discrepancy between N and S halves of a plate. What is the maximum contribution we can expect from them? Suppose they ran all the way from 20^h to 5^h (9 hours of R.A. out of the 24), parallel to the Milky Way and 15° from it. We see from Table XIV. that log N increases 0·17 in 10° of Galactic latitude, or 0·017 in 1°; so that 100 stars on the S half of a plate would become 104 on the N half, due to approach to the Galaxy. This is an increase of 4 per cent., which is about what is required; but it becomes inadequate when we take 9/24 of it in order to distribute it round the whole zone. This fraction 9/24 corresponds to uniform distribution in R.A., which is, of course, not in strict accordance with the facts; but weighting each hour by the number of stars in it gives a nearly equal fraction, since the rich Galactic portions are chiefly in the non-contributing R.A.'s.

(C) For the remainder of the zone contributes nothing, *or goes the other way*. We have spoken of the crossing points in (A) above; from R.A. 5^h–6^h, for instance, the increase northwards is rapid for the first half hour, but is balanced by an equally rapid diminution when the central line is crossed. Reserving for future investigation the effects of possible asymmetry, we may put aside this crossing region as contributing nothing. Proceeding to greater R.A.'s, we are now north of the Galaxy, and thus stars decrease in number as we go northwards. The quantity N–S will thus be negative. But the zone +30° is a small circle of the sphere, and presently its sharper curvature brings it perpendicular to the Galaxy, when the difference N–S will be zero, and then positive again. It is positive for about an hour only (near $12\frac{1}{2}^h$) and then is negative again up to 19^h (say), when we get the other crossing point (19^h–20^h).

21. These facts can, of course, be expressed in tabular form, and are given below in Table XV.; but it seemed desirable to call attention to them also in general terms, because general considerations seem to show that we cannot fairly expect, by improved knowledge of the details, to explain the whole of the difference N–S by actual distribution of the stars in Galactic latitude alone. The figures given below are only approximate: they could doubt-

less be improved, for instance, by using Professor Kapteyn's results in No. 18 of his publications, which have become available by his kindness in sending an advance copy since Table XV. was constructed. But the change would not be great. One of the most important details left outstanding is that of the symmetry or want of symmetry of the Galaxy on opposite sides; but Professor Kapteyn's paper does not touch this point, since he has not yet undertaken any discussion of distribution in Galactic longitude.

Table XV.

R.A.	Gal. Lat. (Zone+30°).	Calcd. N–S. per cent.	Observed (N–S) per cent. 31°.	30°.	29°.	Mean.
0·5	−33°	+3	+ 8	− 6	− 3	0
1·5	−31	+3	+10	−12	+13	+ 4
2·5	−27	+3	+ 8	0	+ 8	+ 5
3·5	−20	+3	+24	−17	+18	+ 8
4·5	−11	+4	+ 8	+30	+34	+24
5·5	0	...	+ 3	+21	+24	+16
6·5	+11	−2	0	−10	− 2	− 4
7·5	+23	−1	+15	− 4	− 5	+ 2
8·5	+35	0	+ 9	− 6	+ 4	+ 2
9·5	+48	0	+ 5	− 1	− 5	0
10·5	+61	0	+ 5	− 3	+ 9	+ 4
11·5	+74	0	+ 4	+10	− 2	+ 4
12·5	+86	+1	+ 4	− 5	0	0
13·5	+79	0	0	+ 3	− 2	0
14·5	+66	0	− 4	+ 5	+ 6	+ 2
15·5	+53	0	+ 8	+11	−11	+ 3
16·5	+40	−1	− 7	+10	+22	+ 8
17·5	+28	−1	+ 1	− 1	+ 3	+ 1
18·5	+16	−2	+ 7	+ 7	+ 3	+ 6
19·5	+ 4	...	+ 2	− 4	+18	+ 5
20·5	− 7	+3	+ 1	+15	+14	+10
21·5	−16	+3	+11	+ 4	+ 9	+ 8
22·5	−25	+3	+ 2	+12	+ 9	+ 8
23·5	−30	+3	+15	+11	+ 8	+11

22. Percentages are given in Table XV. rather than total numbers of stars, in order to avoid undue influence from the Galaxy. It will be seen that the observed percentage is from $20^{h}\cdot5$ to $23^{h}\cdot5$ +9, and from $0^{h}\cdot5$ to $4^{h}\cdot5$ it is +8; the mean for the whole 9 hours being +8·7, as against a theoretical +3·1. For the portion $6^{h}\cdot5$ to $18^{h}\cdot5$ the observed N–S percentage is +2·2, as against a theoretical −0·5. Hence the mean observed N–S is throughout in excess of the theoretical; or, in other

words, only a part of it can be set down to star distribution; the rest must be due either to atmospheric absorption or unsymmetrical performance of the object-glass. But these latter causes do not vary with the R.A. If we estimate their combined effect as 2 or 3 per cent. (being guided by the figures for $6^{h}\cdot5$ to $18^{h}\cdot5$, where the effect of stellar distribution is small), we get 6 or 7 per cent. for the observed effect of stellar distribution in R.A. $20^{h}\cdot5$ to $4^{h}\cdot5$ instead of about 3. This suggests that the rate of diminution with Galactic latitude has been considerably underestimated in those regions. A scrutiny of the figures shows that this is quite possibly the case, the reason being that the rate of diminution varies considerably in different Galactic longitudes. But a complete discussion of this point cannot be given here.

Summary.

§§ 1, 2. Introductory.

§§ 3, 4. Method of measuring the average star density on each quarter-plate, at different distances from the centre.

§ 5. Definition of Δ, which indicates the distance of maximum density from the centre. Table of values of star density and total number of stars for different values of Δ.

§ 7. List of observed values of Δ, in order of date of exposure of plates, in vols. i., ii., and iii.

§§ 8–11. Variations of focal length indicated by changes of mean Δ may be due to discontinuous disturbances of the plate, but are more probably due to seasonal changes in focal length.

But there is certainly one discontinuity, at the erection of the new dome in 1900. Before this the plate was satisfactorily normal to the telescope axis; afterwards it was apparently tilted in R.A., greater R.A.'s being nearer the O.G.

§§ 12–14. In consequence of this tilt, the side of greater R.A. is richer in stars; but the detailed study of the effect of tilt is reserved for a separate investigation.

§ 15. But there is an excess of stars in the N half of the plate, which is 4 per cent. richer than the S half, and this cannot be due to tilt.

§§ 17, 18. Visual atmospheric absorption would give an excess of 0·23 per cent. in the N half: photographic absorption may be greater, but its value cannot be well determined from the Oxford plates, which were taken at nearly constant Z.D. A special investigation of it is desirable.

§§ 19–21. The distribution of the stars in Galactic latitude alone explains a part, but cannot explain the whole of the 4 per cent. excess in the N half of the plates. The general nature of the distribution in Galactic longitude is such as to explain another portion of the excess, but quantitative estimates cannot yet be made.

Further Considerations on the Correlations of Stellar Characters.
By Winifred Gibson, B.Sc., formerly Jessel Scholar, University College, London, and Karl Pearson, F.R.S., University College, London. (With Six Diagrams.)

(1) *Introductory.*—In a paper communicated to the Society last year (*Monthly Notices*, vol. lxvi. p. 445), modern statistical methods were used for the first time to determine the numerical relationships between various star characters. The object of the present paper is to deduce further similar relationships, and to deal with some of the same relationships on the basis of wider data. The general characters with which we have to deal, and which are more or less accurately known for larger or smaller stellar populations, are (1) magnitude, (2) colour, (3) spectral class, (4) proper motion, (5) parallax, (6) position. In any attempt to look upon the stellar universe as an orderly whole, the relationships between these characters must be of fundamental importance. To determine their numerical values is the first stage by which we pass from chaos to an organised and locally differentiated cosmos. The aid which the statistician may venture to offer the trained astronomer in this respect may, perhaps, be illustrated by reference to some recent work. Since the publication of the first paper above referred to, two memoirs, both of considerable importance, have appeared. The first is that of Messrs. Chase, Smith, and Elkin ("Parallax Investigations on 163 Stars, mainly of large Proper Motion," *Transactions, Yale Univ. Observatory*, vol. ii. pp. 1–207). This memoir deals with the relationship of proper motion, magnitude, parallax, and spectral class, among much else of great value, but not bearing on the topics we have at present in hand.

A second memoir of less scope but of considerable interest is that of Mr. W. S. Franks ("The Relation between Star Colours and Spectra," *Monthly Notices*, vol. lxvii. pp. 539–42). Now, these memoirs more than suffice to show that the distribution of star characters is not one of mere random association. The characters occur in a *correlated* manner, and this in itself is suggestive of the cosmos being a differentiated organisation. Even if we at once admit that we might anticipate that parallax would be related to proper motion, or even to magnitude, or, again, that colour and spectral group would be found in association, it is less obvious that spectral class will be found related to magnitude, proper motion, and parallax.

Yet, even when we see these relationships indicated in the above and other memoirs, there appears to be something lacking, which it is, perhaps, possible for modern statistical methods to supply. It would not be possible from the above type of classificatory work to determine the intensity of the relationship between the characters under consideration. For example: Is parallax more closely associated with magnitude or with spectral class? Or, again: Is magnitude more closely related to distance